Shear-enhanced elasticity in the cubic blue phase I

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We present results of the linear and non-linear rheology of cubic blue phase I (BPI). The elasticity of BPI is dominated by double-twist cylinders assembled in a body-centered cubic lattice, which can be specified by disclination lines. We find that the elasticity of BPI is drastically enhanced by applying pre-shear. The shear-enhanced elasticity is attributed to a rearrangement of the disclination lines that are arrested in a metastable state. Our results are relevant for the understanding of the dynamics of disclinations in cubic blue phases.

I. INTRODUCTION

Defects play a crucial role for the rheological properties of soft condensed matter, which shows a pronounced propensity to self-organise into mesoscale structures. Not only the rheology of cholesteric liquid crystalline phases, but also that of the nematic and smectic phase is particularly strongly affected by the dynamics of intrinsic defects [1–8]. Previous studies revealed that the elasticity of liquid crystals is mediated by their defects and that their viscosity is dominated by a creation-annihilation mechanism of defects under shear flow [2, 6, 7]. Defects even affect non-equilibrium phase transitions in the thermotropic smectic phase [8–13]. As defects influence all kinds of rheological phenomena in liquid crystals, it is therefore mandatory to study their dynamics in order to obtain a detailed understanding of fundamental aspects of structural rheology and topological defect networks.

In this study, we focus on the defect-mediated rheology of cholesteric blue phase I (BPI) with the goal to shed some light on the origin of its viscoelastic behaviour. BPI exists in a narrow temperature range as one of the frustrated, intermediate phases between the cholesteric and the isotropic phase [14–19]. In the presence of chiral constituents, the cholesteric host phase forms double twist cylinders (DTCs) as its best compromise between minimising its total free energy and accommodating more twist at the same time. This can be intuitively realised as in DTCs the director field rotates around two perpendicular axes as opposed to only one in a simple cholesteric phase. The DTCs arrange in regular, periodic arrays with characteristic length scales in the range of several hundreds of nanometres. Importantly, DTCs cannot be formed without creating defect regions between them [10]. This frustration gives rise to a network of topological −1/2 disclination lines, which is responsible for many physical properties of BPs, including their rheology and characteristic behaviour under phase transitions. The regular, three-dimensional structure makes them also very suitable for a variety of photonic applications [20–30]. BPs are differentiated into the three sub-phases BPI, BPII and BPIII, depending on the structure and symmetry of their disclination network. BPI and BPII are characterised by a body-centered cubic or simple cubic lattice, respectively [16, 31]. They are easily recognisable under a polarised microscope through their typical platelet texture, which is made up of polygonal crystallites that are separated by grain boundaries. In BPI, the disclination lines are well separated and do not intersect, whereas in BPII they form two interpenetrating lattices that are connected at nodal points and give rise to a tetrahedral network structure. It is generally assumed that these topological differences between BPI and BPII are ultimately responsible for their different response under external strain. BPIII, on the contrary, appears under the microscope as a blue foggy phase that does not exhibit the above mentioned platelet texture [32–34]. The symmetry of BPIII is the same as that of the isotropic phase, which leads to the general conclusion that disclination lines in BPIII form random network without long-range periodicity. The defects destabilise the local order structure, a feature that manifests itself in the strikingly narrow temperature range over which a pure BPIII sample is stable.

BPs exhibit shear-thinning behaviour, just as other liquid crystalline phases, e.g. the smectic phase [9]. Due to experimental intricacies caused by the rather limited temperature range over which BPs are stable, their rheology has been so far only studied numerically. Computer simulations reveal a very rich rheological behaviour, featuring periodic breakup and reformation of the disclination network and its rearrangement under shear [21–23]. The disclination lines can elastically withstand the flow. At low shear rates the apparent viscosity of the BPs is thus significantly higher than that of the isotropic or cholesteric phase. With increasing shear rate the disruption and reconnection of the disclination network entails a periodic oscillatory shear stress on microscopic length scales. Further increase of the shear rate induces a realignment of disclination lines, which leads first to a flow-aligned Grandjean structure, and then upon further increase to a flow-aligned nematic state at the highest
shear rates.

Early studies reported viscoelastic properties in BPs [24, 25]. The detailed rheological behaviour of BPI and BPII was published very recently by Sahoo et al. [26]. They found a viscoelastic, solid-like behaviour of BPI and BPII and identified various flow regimes which qualitatively agree with numerical predictions by Henrich et al. [22, 23]. Interestingly, they find damped oscillations in stress relaxation experiments and the formation of a shear flow-induced Grandjean–cano line. All of these findings are thought to be connected to the realignment, disruption and reconstruction of the disclination network in shear flow. In external electric fields, new BPs with different symmetry have been discovered [35–41]. The electric fields break the cubic symmetry and induce a distortion of the lattice of DTCs. This leads to lower-symmetry phases with orthorhombic, tetragonal, or hexagonal structures. At larger field amplitudes, the regular lattice breaks up and shows a transition to the cholesteric or nematic phase. Such non-equilibrium phase transitions in BPs are expected even under shear flow through a rearrangement of the disclination lines as predicted by Henrich et al. [22, 23].

This work is organised as follows: Section 2 describes the materials and experimental methods. In section 3 we show rheological data and microscopy images of BPI under shear. We concentrate especially on the connection of the disclination network in shear flow. In section 4, we discuss the physical origin of the elasticity of BPI and finally summarise our findings in section 4.

II. EXPERIMENTAL

A. Sample preparation

We used a mixture of four compounds to create the blue phase I (BPI) [22]. As a host liquid crystal, E8, PE-5CNF (4-Cyano-3-fluorophenyl 4-pentyl benzoate), and CPP-3FF (4-(trans-4-uid crystal, E8, PE-5CNF (4-Cyano-3-fluorophenyl 4-

viscosity measurements were performed using a stress-controlled rheometer, MCR-301, Anton Paar, equipped with an Olympus inverted microscope (IX-83). Parallel plate shear cells made of glass with diameter = 30 mm were used for the observations. The sample thickness was kept at 50 µm. The microscopy images under shear were obtained using a CMOS camera, HAS-L1, Ditect Ltd Co. Because of the different light sources in both microscope observations, the typical platelet texture of the BP shows slightly different colours. For the rheological and microscopic measurements no surface treatment was performed. In the cholesteric phase, we observed oily streaks instead of fingerprint textures (data not shown). The anchoring conditions on the surfaces were planar degenerate throughout.

III. RESULTS AND DISCUSSION

A. Identification of BP

Figure 1 shows the temperature dependence of the viscosity η at a fixed shear rate of $\dot{\gamma} = 1$ s$^{-1}$ upon cooling as well as heating. The temperature was scanned in the range between 25.0 and 35.0 °C with a scanning rate of $T = 0.1$ °C/min. The temperature dependence of the viscosity during the cooling process was more gradual and could be measured stably. The viscosity increases rapidly during the cooling process at $T = 32.0$ °C, exhibits a shoulder at around $T = 31.0$ °C and falls off at 29.0 °C. However, the behaviour during heating was not quite as continuous as the viscosity increased steeply at $T = 29.0$ °C, followed by a gradual and slightly unsteady decrease in the temperature range between $T = 29.0$ and 32.0 °C. Furthermore, a significant thermal hysteresis was observed, but only in the cholesteric phase (N*) and below $T = 29.0$ °C. This was also accompanied by different vi-
FIG. 1. Temperature dependence of the viscosity at $\dot{\gamma} = 1$ s$^{-1}$. The different symbols correspond to measurements during the cooling and heating processes. The temperature was changed at a constant rate of $T = 0.1$ °C / min.

usual textures under the polarised optical microscope, which were most noticeable in the quiescent state and became slightly more obscured at low but finite shear rates. The observed temperature dependence of the viscosity is similar to that seen in some other BP systems [20, 26, 43]. It has been previously reported that the same mixture forms a BPI between temperatures $T = 30.4$ and $33.4$ °C [24]. We therefore conclude that the observed increase of the viscosity is due to the formation of the BPI and attribute the pronounced hysteresis in the viscosity of the N$^*$ phase during the cooling process (viz. Fig. 2) to residual effects of the BP structure.

Representative polarised microscopy images of the quiescent state during heating and cooling are shown in Fig. 2. During cooling, the system started in the isotropic phase at $T = 33.0$ °C. The temperature was then decreased at a rate of $T = 0.1$ °C/min. At $T = 32.0$ °C ordered structures are nucleated and grown from the dark, isotropic phase. The size of the ordered regions increases upon further cooling and a different birefringence colour pattern emerges, depending on the orientation of the liquid crystals. At $T = 31.0$ °C spherical coalescing domains of BPI are clearly visible. These ordered domains grow upon cooling and fill the entire sample area at $T = 29.5$ °C. This is a typical feature of BPI and BPII. During the heating process textures different from those described above are observed. The temperature was first set to $T = 28.0$ °C, corresponding to the N$^*$ phase. The temperature was then raised at a rate of $\dot{T} = 0.1$ °C/min. At $T = 29.0$ °C, small platelet-like textures can be seen. The size of the textures increases with temperature, and instead of a multi-coloured, polycrystalline structure a dichromic texture appears. Pre-existing defects in the N$^*$ phase significantly affected this texture during heating. Hence, the two texture of BPI, either grown from the isotropic phase upon cooling or formed from the N$^*$ phase upon heating, are obviously quite different. The hysteresis and unsteadiness that we observe in the viscosity measurements and which we address more specifically below can thus be attributed directly to the distinct textures of the blue phase during heating and cooling.

Following Kleiman et al., Negita, and Sahoo et al. [20, 24, 26] and considering the characteristic platelet texture of the BP, we attribute the sharp increase in viscosity between 29.0 °C and 32.0 °C to the formation of BPI. On the basis of Figs. 1 and 2 the phase transition temperatures in this system can be determined as $T_{\text{Iso/BPI}} = 32.0$ °C for the Isotropic / BPI transition, and $T_{\text{BPI/Chol}} = 29.0$ °C for the BPI / N$^*$ phase transition, respectively. During the cooling process, most of the BPI region is coexisting with the isotropic phase. With regard to the temperature dependence of the viscosity, the shoulder around $T = 31.0$ °C in Fig. 1 can be related to the coalescing BPI domains. The reported phase transition temperatures between the different liquid crystalline phases in calorimetric measurements agree well with those observed in our viscoelastic measurements [24].

B. Thermal hysteresis in the shear moduli

Our BPI system exhibits thermal hysteresis as shown in Fig. 2. In order to confirm this finding from a rheological point of view, the dynamic shear moduli were measured during cooling and heating. The temperature was changed stepwise at a rate of $\Delta T = 0.1$ °C and was kept constant for 600 s so that the system could equilibrate. Then the dynamic shear moduli were measured as a function of angular frequency, followed by the next temperature increment or decrement. This procedure was repeated for temperatures between $T = 32.0$ and 28.5 °C. The shear moduli at representative temperatures are shown in Fig. 3. At $T = 31.0$ °C, corresponding to the BPI/Iso biphase region, storage and loss modulus $G'$ and $G''$ were measured upon cooling and heating. Interestingly, both thermal cycles nearly coincide. $G''$ domi-
FIG. 2. Polarised microscope images during cooling (a) and heating (b) in the quiescent state. The scale bar corresponds to 10 µm. In each process the temperature was changed at a constant rate of 0.1 °C / min.

FIG. 3. Shear moduli $G'$ and $G''$ as a function of angular frequency $\omega$ at different temperatures. The shear moduli measured during the heating and cooling process are compared in the same figure. Filled and open symbols indicate the storage and loss modulus, respectively.

C. Shear hysteresis in the shear moduli

In the following, we present only the experimental data that was obtained for the cooling cycle. As previously demonstrated by numerical studies, the realigning of the network of disclination lines in shear flow can have a determining influence the rheological properties of BPI [21, 23]. Here, we compare the effect of pre-shear on the BPI/Iso biphasic and the homogeneous BPI phase.
FIG. 4. Shear moduli of BPI/Iso biphase at \( T = 31.0 \, ^{\circ}\mathrm{C} \) as a function of angular frequency \( \omega \). Shear moduli measured before and after applying pre-shear rate of \( \dot{\gamma}_{\text{pre}} = 500 \, \text{s}^{-1} \) are compared in panel (a). Solid lines indicate the slope of the shear moduli \( \omega^{-0.5}, \omega^{1.0} \) and \( \omega^{2.0} \) for the reference, respectively. Shear moduli measured at different elapsed times \( \Delta t = 2000 \, \text{s} \) and 4000 s after pre-shear are also shown in panel (b). Filled and open symbols are storage and loss shear modulus, respectively. Solid and dashed lines correspond to the shear moduli after pre-shearing at \( \dot{\gamma}_{\text{pre}} = 500 \, \text{s}^{-1} \).

1. BPI/Iso biphase region

Fig. 4(a) shows the shear moduli in the BPI/Iso biphase region at \( T = 31.0 \, ^{\circ}\mathrm{C} \) before and after the application of pre-shear with a shear rate of \( \dot{\gamma}_{\text{pre}} = 500 \, \text{s}^{-1} \). Without pre-shear the shear moduli \( G^{\prime} \) obeys a power law behaviour like \( G^{\prime} \sim \omega^{1/2} \) in the high frequency domain. We presume this is caused by viscous dissipation between BPI domains that are formed in the isotropic phase as it occurs also for concentrated emulsions \[17\]. The absence of a plateau in the storage modulus \( G^{\prime} \) and the dominance of the loss modulus \( G^{\prime \prime} \) gives evidence that the BPI/Iso biphase behaves like a viscous fluid. After applying a pre-shear at \( \dot{\gamma}_{\text{pre}} = 500 \, \text{s}^{-1} \) the moduli are slightly enhanced in the high frequency domain whilst they still maintain the above mentioned power law behaviour.

The storage and loss moduli in the terminal region are reduced and feature a single relaxation behaviour like \( G^{\prime} \sim \omega^{2} \) and \( G^{\prime \prime} \sim \omega^{3} \), respectively. The modified slope in frequency dependence of the dynamic moduli after pre-shearing indicates that the relaxation dynamics of the BPI is either affected by a realignment of the disclination lines or by the emergence of a polycrystalline texture. In Fig. 4(b) the shear moduli after a specified elapsed time interval are shown. After waiting times of \( \Delta t = 2000 \) and 4000 s the single relaxation-like behaviour remains unchanged, but the shear moduli are smaller and do not recover their initial behaviour. This is consistent with the system adopting a new (meta)stable state. A Maxwellian single relaxation behaviour that we observe after pre-shearing suggests that the platelet size is homogenised during the shearing.

2. Homogeneous BPI

The homogeneous BPI phase shows pronounced viscoelasticity (viz. Fig. 5) contrary to the BPI/Iso biphase system. We found a significant effect of pre-shearing on the dynamic moduli in the homogeneous BPI phase. Pre-shear flow induces an enhancement of the elastic modulus and slows down the relaxation time. The enhanced elasticity does not relax towards that of the original state, but remains high even after the cessation of shear. Instead of the typical platelet texture of BPI, we observe a homogeneous texture in the state with enhanced elasticity. In the following, we show details of the pre-shear effect on the shear moduli.

Before applying the pre-shear, in Fig. 5(a), the storage modulus \( G^{\prime} \) has a plateau modulus at high frequencies. One can also see a second plateau in \( G^{\prime} \) at low frequencies, supporting the existence of a numerically predicted yield stress \[23\]. The loss modulus \( G^{\prime \prime} \) exhibits a minimum around \( \omega = 40 \, \text{s}^{-1} \). For intermediate frequencies at about \( \omega = 10 \, \text{s}^{-1} \) a maximum in \( G^{\prime \prime} \) appears, which is a clear sign of a structural relaxation. The power law behaviour of \( G^{\prime} \sim \omega^{1/2} \) is not observed in the homogeneous BPI. The fact that the modulus does not vanish at \( \omega = 0 \) suggests BPI behaves thermodynamically like a solid phase with a finite yield stress.

Applying pre-shear changes the viscoelasticity of BPI drastically. While the plateau modulus at high
frequencies remains unchanged, the minimum and maximum in $G''$ shift to lower frequencies. At low frequencies the pre-shearing has an even more remarkable influence as the shear moduli are significantly enhanced with increasing pre-shear rate. Particularly after applying a rate of $500 \text{ s}^{-1}$ the storage modulus $G'$ dominates the entire frequency domain. With increasing pre-shear rate the homogeneous BPI phase becomes more elastic and features longer relaxation times.

Fig. 5(b) shows the shear moduli measured for waiting times of $\Delta t = 2000 \text{ s}$ and $4000 \text{ s}$ after the application of the pre-shearing, which gives rise to further change in the shear moduli. The initial state is not fully recovered and the shear-enhanced elasticity that we report here is therefore irreversible. The persistent nature of the dynamic shear moduli indicates that a shear-induced phase has formed in BPI.

The textures under shear flow at different temperatures are shown in Fig. 6. Due to the larger thickness of the sample that is required for rheology experiments the fraction of scattered light is unavoidably larger. This leads to a slightly more blurred appearance of the images. Their polycrystalline structure, however, is still clearly recognisable and demonstrates that we are still operating in the BPI region. Applying a pre-shear rate of $\dot{\gamma}_{\text{pre}} = 500 \text{ s}^{-1}$ at $T = 31.0 \, ^\circ\text{C}$ breaks up the platelet texture in the BPI/Iso biphase, which is then immediately recovered upon cessation of the shear flow. At a lower temperature of $T = 29.5 \, ^\circ\text{C}$ we observe a remarkable irreversibility of the BPI texture as with the same pre-shear applied the texture is completely melted and does not recover.

The polarised microscopy images of BPI at $\dot{\gamma} = 500 \text{ s}^{-1}$ are generally brighter than those of the BPI/Iso biphase. In the BPI/Iso biphase, the dark polarised image under shear flow indicates a collapse of ordered domains. On the contrary, the bright image in the homogeneous BPI suggests a strong orientation of the DTCs or the director field, indicating a structural transition which is induced by the shearing. In the homogeneous BPI, even after cessation of the shear flow the platelet texture is not recovered. The fact that we do not observe any distinct textures like platelets suggests that the nucleation is prohibited and that the BPI is arrested in a metastable state. Here, we would like to remark that a focal conic texture as commonly observed in N* phases is not detected. This gives evidence that BPI is the thermodynamically stable phase and a phase transition to the N* phase is not induced under shear in our experimental condition.

Fig. 7 show the dependence of the storage modulus $G'$ on the pre-shear rate $\dot{\gamma}_{\text{pre}}$ at $\omega = 0.1 \, \text{s}^{-1}$. This frequency has been chosen because we observed for it the most pronounced effect of pre-shearing. The relaxation time $\tau_p$ is estimated as inverse of the frequency of the maximum in the loss modulus $G''$.

At low shear rates $\dot{\gamma}_{\text{pre}} < 1 \text{ s}^{-1}$ the storage modulus $G'_{0,1}$ decreases with $\dot{\gamma}_{\text{pre}}$ and the relaxation time $\tau_p$ remains almost constant. However, above a critical shear rate in the range of $\dot{\gamma}_{\text{pre}} > 0.5 - 1 \text{ s}^{-1}$, $G'_{0,1}$ and $\tau_p$ both increase with increasing $\dot{\gamma}_{\text{pre}}$. Note that $G'_{0,1}$ and $\tau_p$ increase even without exhibiting visible platelet textures, as shown in Fig. 6. This indicates that a
realignment of disclination lines contributes primarily to the slowing down of the relaxation time and thus to the elasticity of the investigated BPI. The findings also suggest that a critical shear rate needs to be applied to trigger the rearrangement process.

**D. Origin of the elasticity**

As previously stated the pre-shearing breaks up the platelet texture at a temperature \( T = 29.5 \) °C, which is also where observe the transition to a homogeneous BPI. Interestingly, the system does not recover the initial state, which is true even after the shear flow has been completely switched off, as shown in Fig. 6. Therefore, the shear-enhanced elasticity has to originate necessarily from the rearrangement of three-dimensional array of DTCs and disclination lines, and not from the re-orientation and compression of the platelet texture. As suggested by numerical studies, the disclination lines can withstand a certain degree of deformation in the shear flow. This observed resistance constitutes at least one possible contribution to the overall elasticity of the BP [21, 23] and may well be the origin of a yield stress.

In Fig. 8 we compare flow curves at different temperatures as well as the second plateau modulus \( G'_{0.1} \) from Fig. 5 and the yield stress. From the flow curves of the homogeneous BPI it is obvious that a yield stress exists, whereas the BPI/Iso biphasic does not exhibit a yield stress. The yield stress \( \sigma_Y \) is almost comparable to the second plateau modulus \( G'_{0.1} \), measured without pre-shear. The correspondence of \( \sigma_Y \) to the second plateau modulus confirms the viscoelastic solid-like properties of the homogeneous BPI.

Following a simple argument [18], it can be shown that the shear modulus scales like \( G_0 \sim k_B T / \xi^3 \), where \( \xi \) has the meaning of a characteristic length scale, such as the lattice constant of a periodic structure. Using the high frequency plateau modulus \( G' \approx 30 \) Pa at \( T = 29.5 \) °C (viz. Fig. 5 left image) this leads to an estimate of \( \xi \approx 50 \) nm. Using the same reasoning and the value of the modulus \( G' \approx 2 \) Pa at the point where it levels off at low frequencies, this yields a value of \( \xi \approx 130 \) nm, which is approximately half the value that has been reported for the lattice constant [18, 31, 49, 50].

Through transmission electron microscopy (TEM), the periodicity of the DTC lattice in BPI was accurately determined to be \( 238 \pm 10 \) nm for the mixture of reactive mesogen RMM-141C (Merck) and the strongly twisting chiral dopant CD-X (Merck) [31]. The dependence of the lattice constant on the chiral dopant for different mixtures has been also studied by reflection spectra and Kossel diagrams, which led to slightly larger values in the range of 280 - 300 nm, showing good agreement between different mixtures [50]. It is interesting that our value of the characteristic length scale \( \xi \) in the rheological experiment is strikingly close to the diameter of a DTC in a BPI lattice and only about a factor two to four smaller than the lattice constant. It should be emphasised that some sort of deviation is hardly surprising, given that the above value of \( \xi \) was obtained by different procedures, i.e. through the dynamical response of BPI. The shift to slightly lower apparent values can also be motivated through the fact that the array of DTCs is not completely rigid, but also subject to a certain degree of deformation during shearing.

Further evidence in support of a characteristic dynamical length scale has been given by Fukuda et al. In their numerical study of the structural forces that are mediated by the BP ordering under both compression and dilation [51] they found that because of the periodicity of BPI the force shows an oscillatory behaviour with minima at a dilation or compression length that equals half a unit cell size of a BPI structure. This corroborates our idea that the length scale \( \xi \) of half a unit cell is indeed the relevant rheological structural unit for the elastic response to shear forces and that the angular frequency dependence of the storage modulus has to originate from the lattice of DTCs. The peak in the loss modulus
$G''$, which appears at frequencies between the low- and the high-frequency plateau regions in the storage modulus $G'$, indicates as well a transition from a high-frequency regime where the dynamical response of DTCs is dominant to another one at lower frequencies that is mainly influenced by the length scale of the disclination lattice. This allows us to interpret the relaxation time $\tau_p$, in our approach estimated through the peak frequency in $G''$, as dynamic signature of the DTCs. The fact that the second plateau modulus corresponds approximately to the yield stress suggests that the distortion of a lattice unit cell of BPI results in the regular array of DTCs and disclination lines being broken up and adopting a shear-induced conformation.

### E. Shear-enhanced elasticity

Above the yield stress, the distortion of the lattice unit cell induces the re-ordering of the disclination lines. But due to the effect that pre-shear has on the measured shear modulus (viz. Fig. 7(b)), the relaxation time $\tau_p$ remains almost constant at low shear rates $\dot{\gamma}_{\text{pre}} < 1 \text{ s}^{-1}$. This insensitivity towards can only be rationalised if no drastic realignment of the DTC and the disclination network takes place.

A more significant rearrangement of the disclination lines occurs obviously above shear rates $\dot{\gamma} > 1 \text{ s}^{-1}$, where both the shear modulus and $\tau_p$ increase, suggesting that the shear-induced state remains arrested in a metastable state and the system does not revert to its original structure. It appears that the peak of $G''$ broadens with increasing pre-shear rate (Fig. 5(a)), which is usually interpreted as a distribution of relaxation times. Hence, the broadening of the peak suggests there is a multitude of metastable states in which the disclination network can get arrested.

Dimensionless Ericksen numbers express the ratio of viscous to elastic forces and are defined as $Er = \eta \dot{\gamma} l / K$ with $\eta$ as shear viscosity, $\nu$ as typical flow velocity, $l$ as typical length scale and $K$ as elastic constant of the liquid crystal. Here, we use $\xi = 130 \text{ nm}$ as characteristic length scale $l$ and estimate $\nu$ as the velocity difference across half a unit cell, i.e. $\nu = \dot{\gamma} \xi$. The value of $\eta$ was taken from experiments at shear rates $\dot{\gamma} > 1 \text{ s}^{-1}$ and assumed to be $\eta \simeq 3.5 \text{ Pa s}$. The elastic constant $K = 3 \times 10^{-12} \text{ N}$ was matched to the value chosen by Meiboom et al. [10]. This leads to an Ericksen number $Er \simeq 0.02$ for shear rates around $\dot{\gamma} = 1 \text{ s}^{-1}$.

In numerical studies a flow regime was found at Ericksen numbers $Er \simeq 0.02$, where shearing leads to periodic break-up and reformation of the disclination network which eventually cause the flow-induced transition of the regular network into an amorphous network state [23]. The disclination network of BPI underwent significant conformational changes even for very low shear rates. Interestingly, the flow-induced amorphous state of the disclination network remained arrested in a metastable state and forms for Ericksen numbers up to $Er \lesssim 4$. These simulations also predicted that the system adopts another flow-induced conformation at high shear rates in the range of the Ericksen number $5 \leq Er \leq 16$, a so-called Grandjean texture, which consists of a cholesteric helix with the helical axis oriented along the direction of the flow velocity gradient. Eventually, at very large shear rates corresponding to $Er > 16$, a transition from the Grandjean texture to a flow-aligned nematic state takes place.

In this experimental study Ericksen numbers were considerably lower in the region of $0.01 \leq Er \leq 0.33$ at $T = 29.5 \degree C$. However, the increase of the shear modulus and relaxation times as a function of the pre-shear rate (viz. Fig. 7) fits well into the general picture of a shear-induced break-up of the disclination network and an amorphous flowing state of BPI. Our polarised microscopy images of the homogeneous BPI at high shear rate, shown in Fig. 6, support this interpretation and are compatible with how an amorphous flowing network of defect lines would appear under the microscope. This suggest there is a shear-induced phase in BPI, similar to those observed in other defect-mediated soft matter systems like thermotropic smectic phases and lyotropic lamellar phases.

### IV. CONCLUSIONS

The shear flow of blue phases is a representative example of structural rheology of soft condensed matter, which is mediated and determined by the dynamics of disclination lines and defects. We studied the rheological behaviour of the cubic blue phase I (BPI). The dynamic shear modulus $G'$ of BPI features a first and second plateau at high and low frequencies. The simple scaling argument for the plateau shear modulus $G_0 \sim k_B T / \xi^3$ suggests that characteristic length estimated from high frequency plateau $\xi \simeq 50 \text{ nm}$ is close to a typical value of diameter of DTC. On the other hand, the length scale obtained from the second plateau at low frequency $\xi \sim 130 \text{ nm}$ approximately corresponds to half the lattice constant in BPI. The structural components responsible for each plateau modulus can be related to the diameter of single DTC and half the size of the lattice unit cell. We also found that the second plateau modulus almost corresponds to the yield stress. This correspondence indicates that the length scale of half the unit lattice cell is relevant rheological structural unit for the elastic response. The origin of the elasticity in BPI can be therefore attributed to the body centered (BC) lattice structure consisting of an array of double twist cylinders (DTCs).

Another interesting finding in this study is that $G'$ of a homogeneous BPI is significantly enhanced by application of a pre-shearing protocol. As the modulus is enhanced with pre-shearing, the relaxation time also slows down. The shear-enhanced elasticity is observed above a pre-shear rate of $\dot{\gamma}_{\text{pre}} \simeq 1 \text{ s}^{-1}$, which corresponds to the Ericksen number of $Er \simeq 0.02$. After
pre-shearing, the platelet texture melts into homogeneous phase without visible texture. No focal conic texture in the shear-induced phase assures that the system is in BPI but not in shear-induced N* phase. The shear-enhanced elasticity above the critical shear rate $\gamma_{pre} \approx 1 \text{s}^{-1}$ stems from the disclination lines that become realigned by the shear flow and then arrest in a metastable state. If the shear-induced phase has an amorphous network of the disclination lines, it will be interesting to compare the rheological behaviour of BPIII, which has a local structure similar to that of BPI. Rheological behaviour of BPIII presented very recently shows similar frequency dependence of $G'$ to that of the shear-induced phase in BPI [43]. Although the chemical components of BPI in this study are different from those of BPIII, the slow relaxation time might be a common feature in both of these systems. If we could compare the detailed rheological properties of BPI and BPIII in the same liquid crystalline system, a structure of shear-induced phase in BPI would be rheologically identified. Although we need further investigation to elucidate the structure of the shear-induced phase, defects play obviously an essential role for the rheological behaviour of cubic BPI.

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